BASIC DESIGN DATA FOR THE USE OF FIBERBOARD IN SHIPPING CONTAINERS

November 1951

No. D1911

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
FOREST PRODUCTS LABORATORY
Madison 5, Wisconsin
In Cooperation with the University of Wisconsin

[illegible] FOR THE USE OF FIBERBOARD[1]

IN SHIPPING CONTAINERS

by

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INTRODUCTION

Since corrugated fiberboard boxes were first used to ship cereals and lamp chimneys about 50 years ago, there has been a continued expansion in their use. Salient advantages, such as low cost, light weight, and ease of packing and unpacking, have contributed to their importance as a shipping container. But there has been a lack of technical information for relating the properties of component paperboard materials to the built-up board and to the finished box. Although much progress has been made in evaluating container strength and some work has been done in technical design, much more needs to be done.

A basic study of fiberboard shipping containers was initiated at the U. S. Forest Products Laboratory before World War II, but discontinued for several of the war years. This early work resulted in methods for evaluating the properties of paperboard and corrugated fiberboard as engineering materials, which showed that the strength properties of the component sheets can be correlated with those of the built-up board. (2, 3, 4, 5, 6, 12, 13, 14)[3]

These conclusions strengthened the premise that fiber containers lend themselves to scientific analysis, and it was concluded that basic design criteria could be developed similar to those which have been developed for boxes made from wood. At the conclusion of the war the study was resumed in cooperation with the Quartermaster Food and Container Institute for the Armed Forces with the specific objective of developing such criteria.

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[1]This paper reports research undertaken in cooperation with the Quartermaster Food and Container Institute for the Armed Forces, and has been assigned number 360 in the series of papers approved for publication. The views or conclusions contained in this report are those of the authors. They are not to be construed as necessarily reflecting the views or endorsement of the Department of Defense.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3]Underlined numbers in parentheses refer to literature cited at the end of this article.

This article discusses procedures for such scientific analysis and gives some basic design criteria obtained since the study was resumed.

Tests and Procedures

Several forms of material and test methods were used in this investigation: (1) component paperboard sheets were subjected to ring-crush, modified ring-crush, and strip-column tests; (2) built-up corrugated boards were given bending, shear, and flexural-shear tests; (3) fiberboard structures consisting of four panels representing a box without top or bottom were given compression tests; and (4) conventional slotted boxes were given compression tests and tested in a revolving drum with can-type loads.

Material, Method of Construction, and Size of Tubes and Boxes

Material.--All of the tubes and boxes tested were made of double-faced corrugated fiberboard. Some of the A- and B-flute board was made on the Laboratory's corrugator and was of balanced construction, having the same basis weight of jute or kraft for both liners in combination with several different corrugating mediums. The liners, which were in thicknesses of either 0.009, 0.016, or 0.030 inch, were combined with either straw, kraft, pinewood, chemfibre, chestnut, or aspen corrugating mediums.

Other boards and boxes included A-, B-, and C-flute fabricated in commercial corrugated box factories. Not all of these boards were of balanced construction, although all were made of paperboard component materials being used commercially. Some of the boxes were made at the Laboratory from V3c and W6c boards produced commercially. Other V3c boxes tested were actually made by a commercial fabricator. Samples of the component materials of all the boards were also tested.

Method of constructing tubes and boxes.--The blank from which each tube was made was cut to the proper size, with the corrugations either perpendicular or parallel to the height dimension, with a sharp power saw. This procedure was employed to secure square top and bottom edges, and to eliminate the damage due to crushing and tearing of the liners and corrugations along the edges, which result when a shearing blade is used for cutting. The joint was made by overlapping one side panel over the edge of the side panel on the opposite end of the blank the thickness of the fiberboard. Cloth-backed tape 3 inches wide was applied to the inside and outside surfaces of the joint to overlap each panel about 1-1/2 inches.

The boxes made and tested were the regular slotted type and had manufacturers joints fabricated by stitching, tape, or glue. The top and bottom flaps were sealed with adhesive

Sizes of tubes and boxes tested --Tubes made of Laboratory boards were made in a variety of sizes from 2 inches square to 35 inches square. The height of the tube varied from 2 inches up to 48 inches.

The tubes and boxes made from commercially-[illegible] were not only square, but oblong cross sections in which the length dimension was either two or three times the width dimension. The perimeters of these tubes and boxes are included within a range of 10 inches to 100 inches. The heights of the tubes and boxes were between 6-1/2 and 36-7/8 inches.

Component Paperboard Tests

Modified ring-crush test.--A modified ring-crush test for paperboard, developed at the Laboratory, was employed to determine the stress at proportional limit, stress at maximum load, and modulus of elasticity of liners and corrugating mediums. A number of layers of the paperboard were rolled into the form of a hollow cylinder, using a special method of fastening the outside lap and of providing flat ends for insertion in the testing machine. Two optical strain gages were attached to each specimen. By measuring compression over a definite gage length on each of two opposite sides of the specimen, reliable data were obtained for use with simultaneous readings of the applied load. Specimens were prepared so that the load could be applied parallel to either the "with-machine" or "against-machine" direction of the paperboard.

Ring-crush test.--Ring-crush tests were made by two methods. One involved testing 1/2- by 6-inch strips supported on the inner surface by one of several removable islands. The islands varied in diameter and were chosen according to the thickness of the specimen. The other method involved tests of strips, 3/8 by 6 inches, supported on the inner surface by a uniform pressure. In both techniques, the strips were [illegible] in such that yielded a single value, that of maximum load.

Strip-column test.--A strip-column test was made on strips 6 inches long that was held straight between two rows of [illegible] to provide a column 1/16 inch high. As with the ring-crush tests, only a single value of maximum load was obtained as a result of applying the load to a testing machine.

Tests of Built-up Board

Bending tests.--The bending test was made on strips of [illegible] board placed across two rolls or bearing supports, which were [illegible] knife edges. The load was applied midway between the supports through a rounded wooden bearing head. Simultaneous readings of load and deflection were taken from the dial of the testing machine and a deflection gage which was located at the middle of the span. Tests were made with the flutes of the board either parallel or perpendicular to the length of the span.

Shear tests.--By gluing a strip of fibreboard between [illegible] blocks and fastening a dial gage to one block and a [illegible] to the opposite block, the relative displacement of the two faces of the fibreboard was measured simultaneously with increments of load which were applied in compression or tension in a universal testing machine. Tests were made with the flutes either parallel or perpendicular to the length of the specimen.

Flexural-shear tests.--By using a test specimen of built-up board in the form of a square flat plate and applying equal loads at the four corners and measuring the mean deflection of the plate, the flexural shearing moduli were determined.

The flexural shearing was accomplished by applying the load downward to two diagonally opposite corners of the plate and upward to the other two. The mean deflection of four points on the diagonals equally distant from the center was read simultaneously with increments of load. The experimental procedure was to plot a load-deflection curve, and from its shape the shear moduli were determined.

Compression Tests of Tubes and Boxes

To determine the relationships of size, shape of cross section, and height involved in the bending and crushing of the side walls, compression tests were made of tubes and boxes. The tests were made in a universal testing machine that had a mechanism for making an autographic load-compression curve of each test. Before test, the tubes and boxes were conditioned in a controlled atmosphere The direction of the flutes in the side walls of the tubes and boxes was either vertical (parallel to the direction of the applied load) or horizontal (perpendicular to the applied load).

Drum tests of loaded boxes.--A number of drum tests were made of various kinds of boxes. They included boards of A-, B-, and C-flute construction with either jute or kraft liners. The boxes had can-type loads and the weight of contents was approximately 1 pound for each inch of length plus depth/ width. Failure was considered complete when the contents spilled from the box during the drum test

Discussion of Results

It was apparent in the initial studies at the Laboratory that the strength properties of corrugated fiberboard and the component paperboard materials could be obtained from tests designed to yield engineering information. In the early tests, the modulus of elasticity of paperboards, as determined by a tension test, was found to correlate with engineering data from column, bending, and shear tests of the built-up corrugated board. It was desired also to determine if a similar correlation could be obtained for compressive stresses. One of the more precise tests was the modified ring-crush test which has previously been described It was found to be useful in correlating the maximum crushing loads of fiberboard tubes with the maximum crushing strength of paperboard Its principal use in this study, however, was to evaluate simpler tests, such as ring-crush and strip-column tests, both of which are considered suitable for use by industry.

Evolution of a Design Formula

One of the main objectives of the investigation was to develop a method of expressing the crushing strength of a corrugated fiberboard box using

information obtained from a simple test of the component paperboard sheets. In the development, the tube was used as the intermediate link between tests of the components and of the box

Before a start could be made toward the consolidation of the mathematical relationships involved in the bending and crushing of the thin plates representing the four walls of a fiberboard tube, some assumptions had to be made. Paperboard being nonisotropic it was recognized that a formula applicable to fiberboard with the machine direction parallel to the applied load might not be applicable to fiberboard with the load applied perpendicular to the machine direction of the paperboard It was also recognized that Fourdrinier paperboard exhibited less difference in strength properties with and across the machine direction of the paperboard than did the jute or cylinder boards With the recognized characteristics as a guide, it was felt that a basic formula could be evolved from data developed at the Forest Products Laboratory for applying the thin plate theory of mechanics to the design of panels of plywood. The plywood is also a nonisotropic material In using these data, the orientation of panel dimensions and selection of axes of reference were as shown in figure 1. This figure represents one of the four faces of a tube Notations used in the various mathematical expressions are explained in the section of this report designated "Notations," and a few of the notations are repeated in the text.

It was found in tests of plywood panels (11) that the stress at which buckling occurred was about equal to the proportional limit of the material, this being about two-thirds of the ultimate compressive strength. From a logarithmic plotting of plywood plates in compression in which the ratio of P (average stress at failure) to P_u (ultimate compressive strength of the material) was plotted as the ordinate and the ratio of P_{cr} (observed critical stress) to P_u as the abscissa, the slope of the curve designated as m was found to be one-third and a good approximation of the data was given by the equation:

$$\frac{P}{P_u} = \left(\frac{P_p}{P_u}\right)^{1-m} \left(\frac{P_{cr}}{P_u}\right)^{m}$$

The problem was to find what relationships would apply in corrugated fiberboard and determine a value of m, the slope of the buckling curve of thin plates

It was found[4] that

$$P_{crs} \text{ (critical stress with shear included)} = \frac{1}{\left(\frac{a^2}{H} + \frac{1}{A}\right)}$$

By substituting the above expression in the equation for plywood, the basic formula is obtained.

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[4] Explanation of the terms of this and subsequent mathematical expressions will be found in the Notations at end of this article.

$$P = \frac{P_p^{1-m}}{\left(\frac{a^2}{H} + \frac{1}{A}\right)^m}$$

Before the proper tests of component sheets and tests of built-up corrugated board had been developed to yield the necessary information, the determination of the unknown quantities of the formula was made from plottings of cubical tubes -- each tube representing four square panels. By the use of bending and shear tests of built-up board, together with the modified ring-crush test of component sheets, it was possible to make these determinations without testing the tubes

Determination of m

To determine the exponent "m," which would adequately describe the shape of a curve representing compression failures of cubical tubes of various sizes, the following method was employed:

From a plotting of cubical tubes, in which the maximum loads, in pounds per inch of panel width, in compression were plotted against size of cube, three points were selected for use. Two of the points selected were representative of the extremities of that portion of the curve in which failure of the tube occurs by buckling The third point selected was midway between the other two. The points were designated as a_a P_a; a_b P_b; and a_c P_c, and selected such that $P_b = \sqrt{P_a P_c}$. The points selected for the original determination of m are designated on a typical plot representing compression failures of cubical tubes (fig. 2).

Substituting the values of these points in the basic formula, three equations were obtained. These equations were simultaneously solved for m and $\frac{H}{A}$.

$$m = \frac{\log \frac{P_a}{P_b}}{\log \frac{a_b^2 + \frac{H}{A}}{a_a^2 + \frac{H}{A}}}$$

$$\frac{H}{A} = \frac{a_a^2 a_c^2 - a_b^4}{2a_b^2 - a_a^2 - a_c^2}$$

The value of the exponent used in the initial calculations was determined as 0 335. The value, 0.272, was used for calculations involving loads applied perpendicular to the flutes

Because it had been seen by trial calculations that slight differences in the value of m did not appreciably change the total calculated tube load, it became apparent that average values of m could be used and thus eliminate the necessity of making determinations of m for each specific board in future calculations Based on the determinations involving several boards, the average values of m were found to be 0.376 and 0 253 for tubes with the load applied parallel and perpendicular to the flutes, respectively. However, the values 1/3 and 1/4 were chosen to simplify calculations.

Determination of A, Transverse Shear Stiffness Factor of a Panel (8, Equation 17)

$$A = h\left[\mu_{yz} + \mu_{xz}\frac{b^2}{n^2\ a^2}\right]$$

The integer "n" (number of half waves that a panel shapes itself into under stress) is so chosen to make the sum of $D_1 \frac{b^2}{n^2\ a^2} + D_2 \frac{n^2\ a^2}{b^2}$ a minimum. The values of shear moduli were determined independently by shear tests of built-up board, using the formula

$$\mu = \frac{P \cos \theta}{WL} \times \frac{c}{d} \quad ,$$

where

- μ = shear modulus
- P = shear load of a built-up board at 0.001-inch displacement of faces, pounds
- θ = the angle of plane of specimen surface with the plane of loading platens, used as 90°
- W = width of specimen, inches
- L = length of specimen, inches
- c = thickness of core, inches
- d = displacement of faces, used as 0.001 inch

With the shear modulus known, A can be determined from the formula

Determination of H, the Bending Stiffness Factor of a Panel

From the plotting of cubical tubes, $\frac{H}{A}$ was known, and with the solution of A in the preceding paragraphs, H could be determined by the relationship $H = \frac{H}{A} A$.

As an alternate method, H can be determined from EI, determined by bending tests or column tests of built-up board, by the following equation (8, Equation 19):

$$H = \pi^2 \quad D_1 \frac{b^2}{n^2 a^2} + D_2 \frac{n^2 a^2}{b^2} + 2K$$

in which (10, Equations 63 and 83)

$$D_1 \text{ or } D_2 = \frac{(EI)_T}{1 - (EI)_T \frac{1.5c}{\mu h \lambda_f} \frac{12}{h^3 - c^3} \left(1 - \frac{c^2}{h^2}\right) \frac{h^2}{L^2}}$$

The value of K can be determined by several methods for use in the foregoing formula for determination of H. It can be determined approximately by the formula $K = \sqrt{D_1 D_2}$, or by another approximate formula using data from plottings of cubical tubes, in which

$$K = \frac{H - \pi^2 \quad D_1 \frac{b^2}{n^2 a^2} + D_2 \frac{n^2 a^2}{b^2}}{2\pi^2}$$

A more accurate test method of determining the flexural shear factor, which had been developed for measuring the shearing moduli in wood (9), was used to determine the flexural shear factor of built-up corrugated fiberboard. It is

$$G = \frac{3u^2}{2h^3} \frac{P}{W}$$

where

P = load applied to each corner of plate
h = thickness of plate
G = shearing modulus associated with a shearing strain referred to axes X and Y. For a point on a diagonal at a distance u from the center, $X = Y = \frac{u}{\sqrt{2}}$
W = width of specimen in inches

Because the values of shear modulus determined by either the approximate method, $K = \sqrt{D_1 D_2}$, or the above formula were found to be comparable, the approximate method was employed in later calculations for simplicity.

Determination of $\underline{H}$ from Modulus of Elasticity

The results of the modified ring-crush test, which can be used for calculating the modulus of elasticity of paperboard in compression, were used to determine the $\underline{D_1}$ and $\underline{D_2}$ in the $\underline{H}$ formula. The true bending $\underline{EI}$, $\underline{D_1}$ and $\underline{D_2}$, may be calculated from the equations:

$$D_1 = \frac{h^3 - c^3}{12} E_{fx} \quad \text{and} \quad D_2 = \frac{h^3 - c^3}{12} E_{fy} + \frac{c^3}{12} E_c$$

where

$$E_c = E_m \frac{A_m}{c},$$

in which

E_m = modulus of elasticity of corrugating material in the across-machine direction

A_m = cross section area per inch of width = $\frac{t(c - t)}{Sk} E(K_1 \frac{\pi}{2})$,

where

$$k = \sqrt{\frac{\pi^2 (c - t)^2}{4S^2 + \pi^2 (c - t)^2}} \qquad \text{and}$$

$E(K_1 \frac{\pi}{2})$ = an elliptic integral determined from tables when $\underline{K}$ is known

Determination of $\underline{P_p}$ from Curve Data of Cubical Tubes

Before the modified ring-crush test was used, the proportional limit of built-up corrugated board was determined from plottings of cubical tubes The method described under "Determination of $\underline{m}$" was used, solving the three simultaneous equations for the proportional limit value $\underline{P_p}$

$$HP_p^{\frac{1-m}{m}} = P_a^{\frac{1}{m}} (a_a^2 + \frac{H}{A}) = P_b^{\frac{1}{m}} (a_b^2 + \frac{H}{A}) = P_c^{\frac{1}{m}} (a_c^2 + \frac{H}{A})$$

The value of $\underline{H}$ being known, and the values of $\underline{m}$ and $\frac{H}{A}$ being determined by use of the simultaneous equations, the value of $\underline{P_p}$ can be computed by use of any one of these expressions.

Determination of P_p from Modified Ring-crush Test

By use of the modified ring-crush test of paperboards, the values of proportional limit, maximum crushing strength, and modulus of elasticity in compression are obtained. From this information, the P_p of the built-up fiberboard can be calculated by:

$$P_p = P_{pf} (h - c)a + P_{pc} \alpha a t c$$

Reasonably accurate predictions of crushing strength of tubes were obtained with the use of the basic formula. The calculations and manipulations involving values from tests of both paperboard and the built-up corrugated board, however, were deemed too complicated for practical use. As a result, simplification of the procedure of making predictions of compressive strength was started.

It was apparent that the first step in simplification was one of finding a simple test of the component paperboard sheets. Secondly, it appeared advisable to eliminate, if possible, the tests of built-up board involved in the procedure.

A search was started for a simple test of the components, the resulting values of which could be combined in the proper proportion to provide an index for the compressive strength of the built-up board. A significant finding at this stage of the development provided the basis for considerable progress in simplification. It was found that the combined ring-crush value designated P_x (pounds per inch) of different boards corresponded to the compressive strength of a specific size of cubical tube when the crushing load was applied parallel to the flutes and to another size when the crushing load was applied perpendicular to the flutes. The two values were found to be constant (designated a_{x2} values) for tubes made of various combinations of materials. A different a_{x2} value was found, however, for A-, B-, and C-flute construction. From actual tests of tubes made from a variety of boards, these values were determined to be 8.36, 5.00, and 6.10 for A-, B-, and C-flutes, respectively, when the crushing load was applied parallel to the flutes. With the establishment of this relationship the final step in the evolution, that of relating the tube to the finished box, could be started.

Relationship of Static Tube and Box Loads

It has been pointed out that the static compressive strength of tubes represents the optimum that may be obtained with any given corrugated board, and it follows that these optimum compressive loads will not be attained in corresponding corrugated fiber boxes because of various factors that enter into their manufacture and use. Hence, in order to use for design purposes the formula which had been developed for the tube, it was necessary to establish the relationship between the tube and box.

To determine the relationship of the crushing strength of fiberboard tubes to the top-to-bottom crushing strength of the finished fiberboard box, comparisons of tube and box loads for corresponding sizes were made. From the comparisons, which included those with square and oblong cross sections in various heights, it was observed that the relationship was fairly constant up to certain limits. For instance, for tube loads up to about 1,500 pounds, the box loads were approximately 0.7 of the corresponding tube loads. For tube loads greater than 1,500 pounds the ratio of box loads to tube loads was no longer constant but decreased with an increase in tube load. It was observed that the tube loads continued to increase beyond the 1,500-pound value while the corresponding box loads did not change appreciably.

In general, the maximum load that tubes of a given cross section will withstand decreases with an increase in height from 2 inches up to 12 or 16 inches, depending upon the kind of material from which it was made. A further increase in height had little influence on its resistance to crushing. This can be explained by the fact that the lower limit of length, that is, the 12-, 14-, or 16-inch length, represents the wave length into which any particular combination of material would shape itself under stress and that the greater lengths were merely multiples of this wave length. Increases in compression strength did not occur, however, with decreases in height of boxes. This can be accounted for, at least in part, by the end condition of the side panels of the box. Due to the horizontal score, which has been found to be one of the weaker points in a box, rolling and bending takes place along the score, usually resulting in premature buckling. As a result, the higher loads are not attained by the shorter panels as they are with a tube where normal buckling occurs. Although some differences in loads were attained for boxes of various heights, for practical purposes, the box loads have been considered the same for a specific cross section regardless of height.

Application of Formula to Box

It was found that the ratio of box load to tube load (box factor) for various cross sections with heights 12 inches and greater was reasonably constant. For heights less than 12 inches, however, there was considerable divergence between the box and tube loads. This was due to increases in tube loads for decreasing heights while the box loads remained about constant throughout the range. Therefore, to eliminate this divergence by deriving a box factor that would apply regardless of the shape of the box, it was necessary to relate the box to a tube having an a/b ratio $\left(\frac{\text{width of panel}}{\text{height of panel}}\right)$ of 1.5 or less. Although it was found that a single box factor could be used for a specific flute, the same factor could not be used for all three flutes. Hence, box factors were determined for A-, B-, and C-flute boxes. Further, it was found that the box factor provided a means for adjusting box loads for the specific kinds of body joints. Some tentative box factors which have been determined are included in table 1.

Table 1.--Tentative box factors for A-, B-, and C-flute boxes

Source of boxes	Type of manufacturer's joint	Box factors (J) for boxes with flutes vertical in side walls[1]		
		Flute		
		A	B	C
Laboratory made from commercial material	Taped	0.717	0.752	0.717
	Stapled	.....	.622	.....
Commercially made	Taped	.677	.597	.667
	Stapled	.....	.564	.....

[1]Box factors for boxes with flutes horizontal in side walls have not been determined.

As a result of the establishment of a_{x2} values and box factors, the design formula for calculating the top-to-bottom compressive strength of the finished corrugated fiberboard box with flutes vertical in side walls now becomes:

$$P = P_x \left(\frac{a_{x2}^2}{\left(\frac{Z}{4}\right)^2} \right)^{1/3} ZJ$$

in which

P = total compressive strength of box in pounds
P_x = composite ring-crush load of built-up board (pounds per inch) ($P_{r\ell}$ single face + $P_{r\ell}$ double back + $\alpha \times P_{rc}$)
a_{x2} = either 8.36, 5.00, or 6.10 for A-, B-, or C-flute, respectively
Z = perimeter of box in inches
J = box factor

Alinement Charts

To simplify use of the formula, alinement charts for calculating the strength of A-, B-, or C-flute boxes have been constructed (figs. 3, 4, and 5). To use the alinement charts, determine the combined ring-crush strength of the single-face liner (S.F.L.), the double-back liner (D.B.L.), and the corrugating medium (C.M.) For A-flute boxes apply the formula

$$S.F.L. + D.B.L. + 1.523 \times C.M.$$

using a tentative box factor of 0.667 for commercially made boxes with taped manufacturer's joints.

For B-flute boxes use

$$S.F.L. + D.B.L. + 1.361 \times C.M.$$

with a tentative box factor for commercially-made boxes with taped joints being 0.597 and those with stapled joints, 0.564

For C-flute boxes use

$$S.F.L. + D.B.L. + 1.477 \times C.M.$$

with a tentative box factor of 0.667 for boxes with taped joints. Then, using a straightedge, connect appropriate point A with box perimeter at point C. With point B as a pivot, orient the straightedge with the box factor, point E and read the load on the compressive strength scale at point D.

Determination of Stacking Strength

It has been known that corrugated fiberboard boxes cannot be expected to support a stacking load equivalent to the load attained by a compression test of the box in a testing machine. But, although some large users of fiberboard containers have established their own stacking limits for boxes in storage (1), what has not been generally known is the portion of the compression test value the box can be expected to support for specific periods of time in various storage atmospheres.

To determine the information deemed necessary for establishing load limits for specific periods of storage, long-time loading tests were made of several kinds of A- and B-flute boxes in several different atmospheres (7). The results thus far obtained have indicated clearly defined trends and relationships between the machine compression test value of boxes, the magnitude of the dead load of storage, and the duration of loading.

Compression Tests

The top-to-bottom static compression test value of the finished container when empty was used as a basis for determining the amount of dead load to apply in the duration-of-load tests. The dead loads were portions ranging from 55 to 95 percent of this static compressive strength. The static compression test values were determined by tests of similar boxes in the various atmospheres in which the duration-of-load studies were to be conducted.

Periods of Reaction

The behavior of corrugated fiberboard boxes subjected to various "dead loads" appeared to follow a general pattern as shown by the reactions during three distinct periods of time. The first period, in which there was a rapid

compression of the boxes, resulted from the initial application of the load, and started the instant the load contacted the box. Some of the rapid compression can be attributed to flattening of the rounded portion of the score along the horizontal edges of the box, together with a general leveling of the surfaces. The rapid compression continued, but at a decreasing rate for a comparatively short period of from a few seconds to 1 to 2 hours, with a rather abrupt transition into the second period. The compression in the second period continued at a uniform but much slower rate. Compression in the third period increased more and more rapidly until failure occurred. The three periods described above were found to exist for all dead load durations whether for a few minutes or 30 days or more, the only significant difference being in the slope and length of the linear portion, period 2, of the resulting curves.

Generally, during the third period, failures were complete and included buckling and crushing of all four panels. The typical box had two of the four panels bowed in and the other two bowed out.

Relation of Load to Duration

When the dead loads represented a fairly large percentage of the compression test values, slight changes in the amount of dead load applied to a box changed the duration considerably. Loads that approached the static compressive strength of the box caused failures usually within minutes. Dead loads which were about 60 percent of the static compressive strength extended the duration to about a month. An example of the relationship between the load and duration may be seen by the actual test results of four typical boxes included in the following tabulation:

Static compressive strength of comparable box	Actual dead load on box	Ratio of dead load to static compressive strength	Time to failure
Pounds	Pounds	Percent	Minutes
702	664	95	1.3
699	610	87	7.3
696	544	78	399.0
696	403	58	35.6 (days)

Duration-of-load tests in which the dead loads approached the static compressive test value of the boxes are shown on the curve, figure 6, by the points in that portion of the curve marked A-B. In the same figure the static compressive strength of the boxes representing the 100 percent level is shown at A^1 at a duration of about 1-1/2 minutes.

The curve of figure 6 may be used to determine the time to cause failure of any specific box regardless of size, magnitude of strength, and moisture content of the fiberboard, when any specified amount of dead load is applied. The reason is that the curve, which is based on the ratio of the dead load to the static compressive strength, expresses a relationship which was found to apply

for all materials and all moisture conditions studied. Knowing the ratio for a specific set of conditions figure 6 may be entered at the appropriate percentage level and the point of entry projected horizontally to intersect the curve. The duration, expressed as days, may be read on the opposite scale. The straight line portion, B-C, of the curve in figure 6 shows that for each decrease of about 8 percentage points in the ratio of the dead load to the static compressive strength the duration of load to cause failure is increased about eight times.

It must be pointed out that the boxes included in this study had not been previously loaded or roughly handled before being used in the duration-of-load tests. A box which is damaged as a result of rough handling prior to storage would not be expected to sustain the same dead load for the same duration of time as an undamaged box. Also, the length of time a box can be expected to sustain a dead load will be reduced as a result of increases in moisture content of the fiberboard during storage. This became apparent from the static compressive test values of boxes in various atmospheres.

Relation of Moisture Content of Fiberboard to Compressive Strength

The influence of moisture content on compressive strength of four lots of boxes, made of different materials, are shown in the curves of figure 7. Here it is seen that the different curves have about the same slope, and for practical purposes it would appear that an average slope represented by the broken line could be used. Using the representative broken line curve, a formula expressing the relationship of compressive strength of boxes to moisture content of the fiberboard was derived to facilitate the use of the data. The formula was derived as follows:

(1) The broken line curve represents a box that has a compressive strength of 1,516 pounds at 0 percent moisture content.

(2) The strength of other boxes at all moisture content values will be represented by parallel lines intersecting X = 0 at various compressive strengths.

(3) This may be expressed by

$$Y = b(10)^{mx}$$

in which

Y = compressive strength of box-pounds
b = compressive strength at 0 percent moisture content
m = average slope (determined to be -3.01)
x = moisture content[5]

[5]For purposes of this equation, moisture content must be expressed as a decimal, determined by dividing the weight of water by the oven-dry weight of fiberboard.

(4) The compressive strength of a box at a specific moisture content may be found by relating the box to one for which the compressive strength and moisture content are known, thus:

$$P = \frac{P_1 \, (10)^{3.01x_1}}{(10)^{3.01x_2}}$$

in which

P = compressive strength to be determined-pounds
P_1 = known compressive strength-pounds
x_1 = moisture content for box having P_1 compressive strength
x_2 = moisture content of box for which the compressive strength is to be determined[5]
3.01 = slope of curve

For easier use of the relationship, an alinement chart was constructed from which the compressive strength of boxes at one moisture content can be readily interpreted in terms of another (fig. 8). To use the chart, connect points A and C, using a known compressive strength for a box at a specific moisture content, with a straight edge. With point B as a pivot, orient the straight edge to the moisture content, point E, for which the corresponding compressive strength is desired, and read the load on the compressive-strength scale at point D. The example indicated by the lines on the chart shows that the compressive strength of 1,000 pounds for a box at 6 percent moisture content is reduced to 430 pounds when the moisture content is increased to 18 percent.

Comparison of Compressive Strength of Boxes with Resistance to Rough Handling in the Revolving Drum

Although it had been anticipated that general trends could be established, a close correlation between compressive strength and the results of rough-handling tests in the hexagonal drum was not expected. Past experience had shown that the drum test is less precise than the compression test, and that test values vary more for boxes tested in the drum than for those tested in the compression machine. Some relationship, however, was found to exist.

Generally, the boxes that attained the greatest compressive loads also attained the greatest number of falls in the drum. For example, the V3c boxes were stronger in compression than any of the boxes tested, and they attained about three times as many falls as the next best box. In comparing some W6c boxes, including 10 sizes and shapes, with similar V3c boxes, the following relationship was observed: the average compressive strength of the W6c boxes was 578 pounds and the average number of falls in the drum to cause failure was 170. For V3c boxes the values were 1,011 pounds and 570 falls.

Although more data are needed, it is felt that eventually it may be possible to obtain some general correlation between results of drum tests and compression tests in considering performance standards for corrugated fiberboard boxes.

How Can Basic Design Data Be Used

It is intended that the information obtained in this study of fiberboard and the basic component paperboard sheets will be used:

(1) To prepare tables and charts for design purposes and general specifications applicable to various box sizes, load limits, and perhaps commodity classifications.

(2) To develop design criteria that can be used by the box manufacturer in quality control operations as well as for design purposes to meet specific use requirements or standards.

The use of the basic information obtained in this study can best be illustrated by the solution of a hypothetical problem.

Let it be assumed that a regular slotted B-flute corrugated box having vertical flutes in the side walls is needed for a specific use, and the box must satisfy the following requirements:

(1) The box must be 12 inches high and the perimeter is to be 82 inches.

(2) The weight of the article and box is to be 40 pounds.

(3) The box is to be constructed with a taped body joint.

(4) The box with packaged article will be in storage 60 days before it reaches the consumer.

(5) The boxes will be piled seven high in the storage warehouses.

(6) Moisture content of the fiberboard during the storage period might be as high as 18 percent.

The problem is one of selecting the proper component paperboard sheets having the desired strength properties from which to fabricate the board for the box.

The solution to the problem will be made with the use of charts contained in this article as follows:

(1) Boxes, each weighing 40 pounds, piled seven high will place a 240-pound dead load on the bottom box of each pile for a period of 60 days.

(2) It may be seen from figure 6 that for storage involving 60 days the dead load must not exceed 56 percent of the top-to-bottom compressive strength of the box. Hence, the compressive strength of the box having 18 percent moisture content needs to be 430 pounds (240 pounds divided by 0.56 = 430 pounds).

(3) From figure 8, it may be seen that a box having a compressive strength of 430 pounds at 18 percent moisture content has a compressive strength of 760 pounds at 9.8 percent moisture content. (The latter condition was employed for development of design criteria presented in this study.)

(4) It may be seen from the solution drawn on figure 4 that a board having a combined ring-crush strength of 39 pounds per inch (across-machine direction) is needed to satisfy the conditions involving the 82-inch perimeter box, 12 inches high, which has a compressive strength of 760 pounds when the moisture content is 9.8 percent.

From the partial inventory of paperboard stock included in the following tabulation the materials may be selected. It may be seen that liners Nos. 1 and 2 can be used with corrugating medium No. 1. Also liner No. 3 could be used for both single-face and double-back with corrugated medium No. 2, but the resulting box would be stronger than necessary. Therefore, a B-flute board made from 47-pound Fourdrinier kraft liners with chemfibre corrugating medium will be fabricated into a box that will meet the requirements of the problem.

Inventory of Materials

Material	Basis weight	Ring-crush strength across-machine direction
	Lb. per 1,000 sq. ft.	Lb. per in.
Liners		
1 Fourdrinier kraft	47	15.78
2 Fourdrinier kraft	47	15.33
3 High-density kraft	48	16.10
4 Jute	52	10.33
5 Jute	52	9.64
6 Fourdrinier kraft	40	12.21
7 Fourdrinier kraft	37	9.81
8 Jute	56	11.98
Corrugating Mediums		
1 Chemfibre	28	5.97
2 Semichemical	26	7.22
3 Bogus	26	6.97

From the solution of the hypothetical problem it becomes apparent that the information contained in this article can be applied to the inventory of any commercial manufacturer. The manufacturer, likewise, can design boxes to meet the requirements of his clientele on the basis of a simple ring-crush test of the paperboard sheets.

Notations

The choice of axes and direction of applied load in relation to flutes are shown in figure 1.

1. a = width of panel in inches.
2. α = ratio of length of corrugating medium when flat to length when corrugated (A-flute = 1.523, B-flute = 1.3605, C flute = 1.477.)
3. a_{x2} = average width, in inches, of cubical tube corresponding to load $P_x = a \sqrt{\left(\frac{P}{P_x}\right)^{\frac{1}{m}}}$
4. b = height of panel in inches
5. c = thickness of core (distance between liners), in inches.
6. A = transverse shear stiffness factor of a panel
7. D_1 = true bending EI of a panel in a direction perpendicular to the applied crushing load.
8. D_2 = true bending EI of a panel in a direction parallel to the applied crushing load
9. E_c = modulus of elasticity of the core in a direction parallel to the flutes
10. E_{fx} = modulus of elasticity of paperboard liners in a direction parallel to the X-axis.
11. E_{fy} = modulus of elasticity of paperboard liners in a direction parallel to the Y-axis.
12. EI = product of modulus of elasticity and moment of inertia of built-up corrugated board.
13. EI_T = product of modulus of elasticity and moment of inertia of built-up board from bending tests, per inch of width, in a direction parallel or perpendicular to the direction of the flutes (dependent upon P.
14. h = thickness in inches, of built-up corrugated board
15. H = bending stiffness factor of a panel.
16. J = box factor, ratio of box load to tube load - dependent upon flutes and type of body joint.
17. K = flexural shear stiffness factor.
18. L = length of span, in inches.
19. λ_f = 1 - (Poisson's ratio with the machine multiplied by Poisson's ratio across the machine direction of paperboard). Note: Average Poisson's ratio of two Fourdrinier kraft paperboards "with machine" = 0.328, average "across machine" = 0.219. Hence, 1 - (0.328 x 0.219) = 0.928.
20. m = an exponent in the compression formula describing the slope of buckling curve for thin plates.
21. n = an integer (number of half waves into which the panel shapes itself under stress and is so chosen to make the quantity

$$D_1 \frac{b^2}{n^2 a^2} + D_2 \frac{n^2 a^2}{b^2} \quad \text{a minimum).}$$

22. P = maximum compressive load of a panel whose width is a, in pounds per inch of width, and whose height is b, in inches.

23. P_p = compressive load at proportional limit of built-up corrugated fiberboard in pounds per inch of width. Note: This value may be for flutes parallel or perpendicular to load, dependent upon P desired.

24. P_{pc} = compressive proportional limit load in pounds per square inch in the with- or across-machine direction of corrugating material, dependent upon P. Note: When the load of a panel with flutes parallel to the load is to be calculated, then P_{pc} is regarded as in the across-machine direction of the corrugating, but when the load is applied perpendicular to the flutes this quantity is disregarded.

25. P_{pf} = compressive proportional limit load in pounds per square inch, in the with- or across-machine direction of the liner material, dependent upon P.

26. P_u = ultimate compressive load of built-up board, in pounds per inch.

27. P_x = ring-crush load in pounds per inch of built-up board determined from summation of the loads of the liners and corrugating medium.

28. $P_{r\ell}$ = ring-crush load in pounds per inch of a 1/2- by 6-inch strip of liner either with or across the machine direction, dependent upon P.

29. P_{rc} = ring-crush load in pounds per inch of a 1/2- by 6-inch strip of corrugating medium in the across-machine direction.

30. S = half the distance, in inches, from crest of one corrugation to the next.

31. t = thickness of corrugating material, in inches.

32. μ_{xz} = shear modulus in a direction perpendicular to load.

33. μ_{yz} = shear modulus in a direction parallel to load.

34. Z = perimeter of box, in inches.

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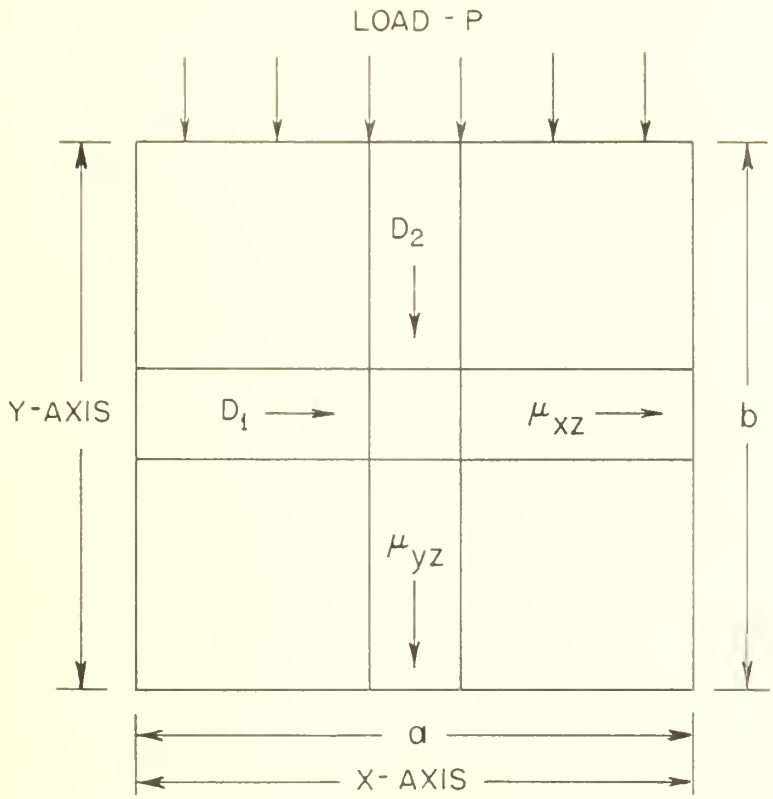
Figure 1.--A panel of fiberboard representing one of the four faces of a tube, showing orientation of dimensions, loads, and stresses.

(ZM 88065 F)



Z M 88065 F

Figure 2.--Typical curves of calculated and actual compression test loads for A-flute cubical tubes made from corrugated board having 0.016-inch Fourdrinier kraft liners and 0.009-inch Fourdrinier kraft corrugating medium (typical data used in determination of m).

(ZM 88066 F)

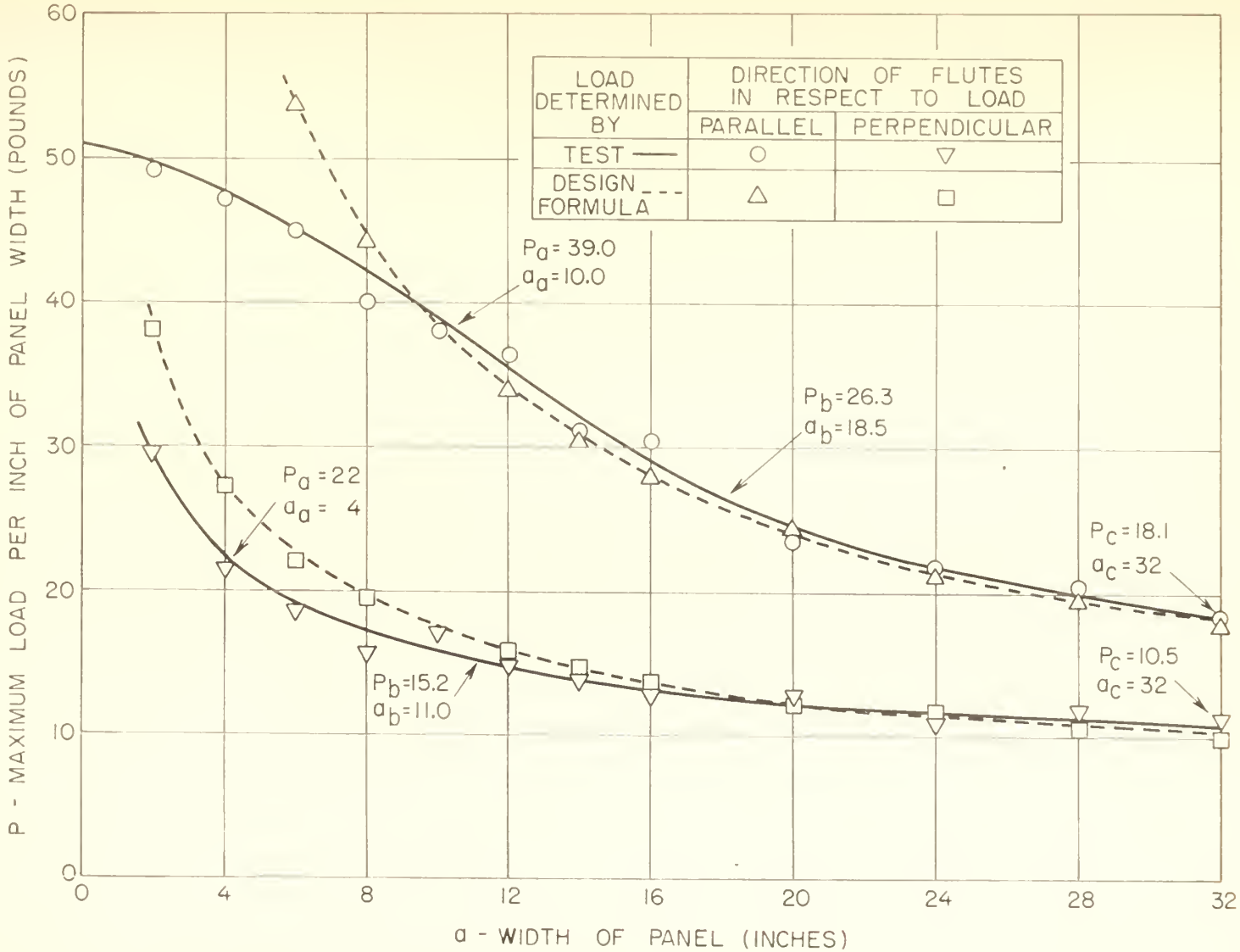
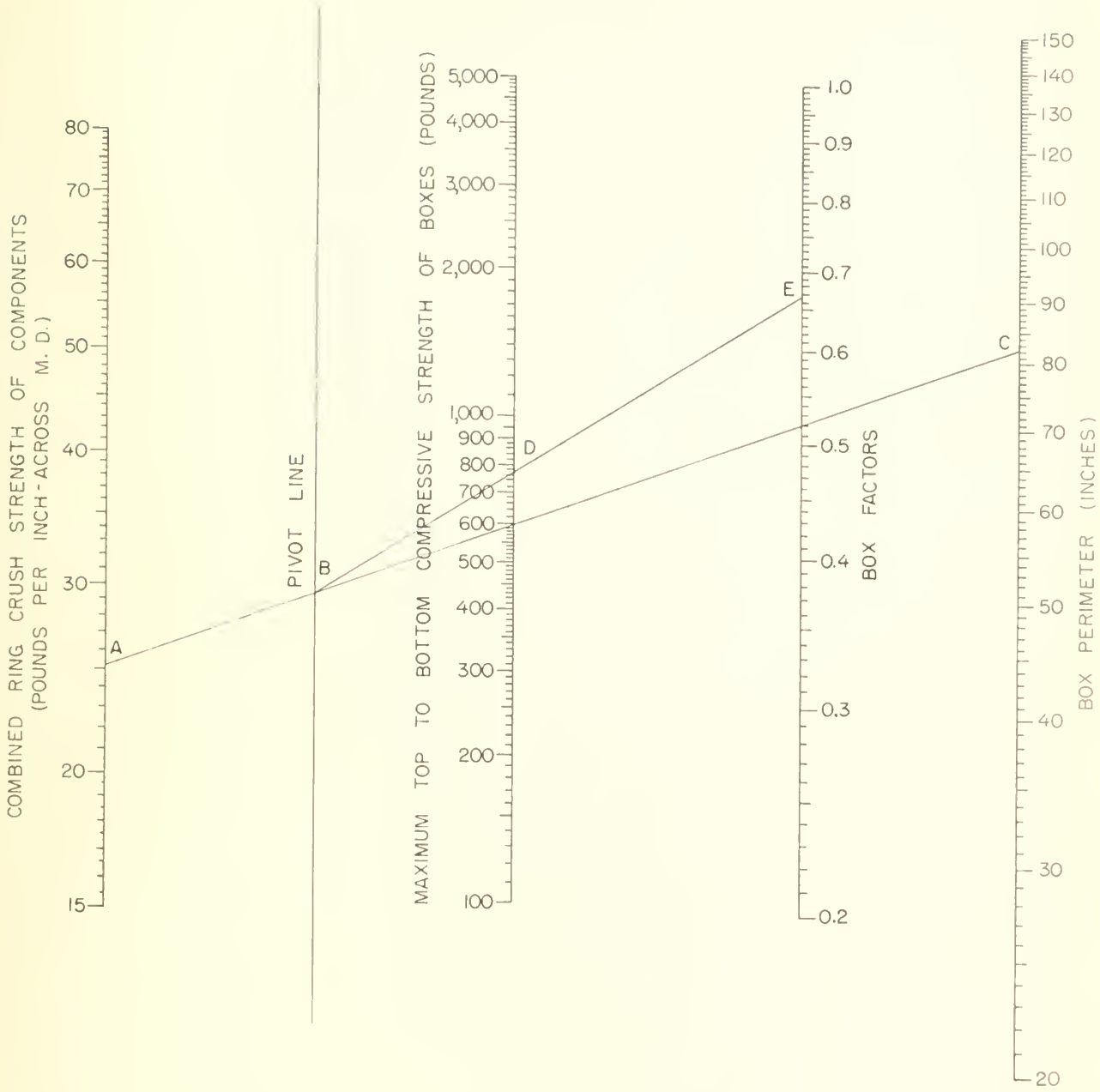

LOAD DETERMINED BY
DIRECTION OF FLUTES IN RESPECT TO LOAD
PARALLEL
PERPENDICULAR
TEST
DESIGN FORMULA
$P_a = 39.0$
$a_a = 10.0$
$P_b = 26.3$
$a_b = 18.5$
$P_c = 18.1$
$a_c = 32$
$P_a = 22$
$a_a = 4$
$P_b = 15.2$
$a_b = 11.0$
$P_c = 10.5$
$a_c = 32$
P - MAXIMUM LOAD PER INCH OF PANEL WIDTH (POUNDS)
a - WIDTH OF PANEL (INCHES)
0
10
20
30
40
50
60
0
4
8
12
16
20
24
28
32

Figure 3.--Chart for determining the top-to-bottom compressive strength of A-flute corrugated fiberboard boxes when the flutes are vertical in the side walls. The resulting determinations will be for a box conditioned in the same atmosphere in which the ring-crush values were obtained.

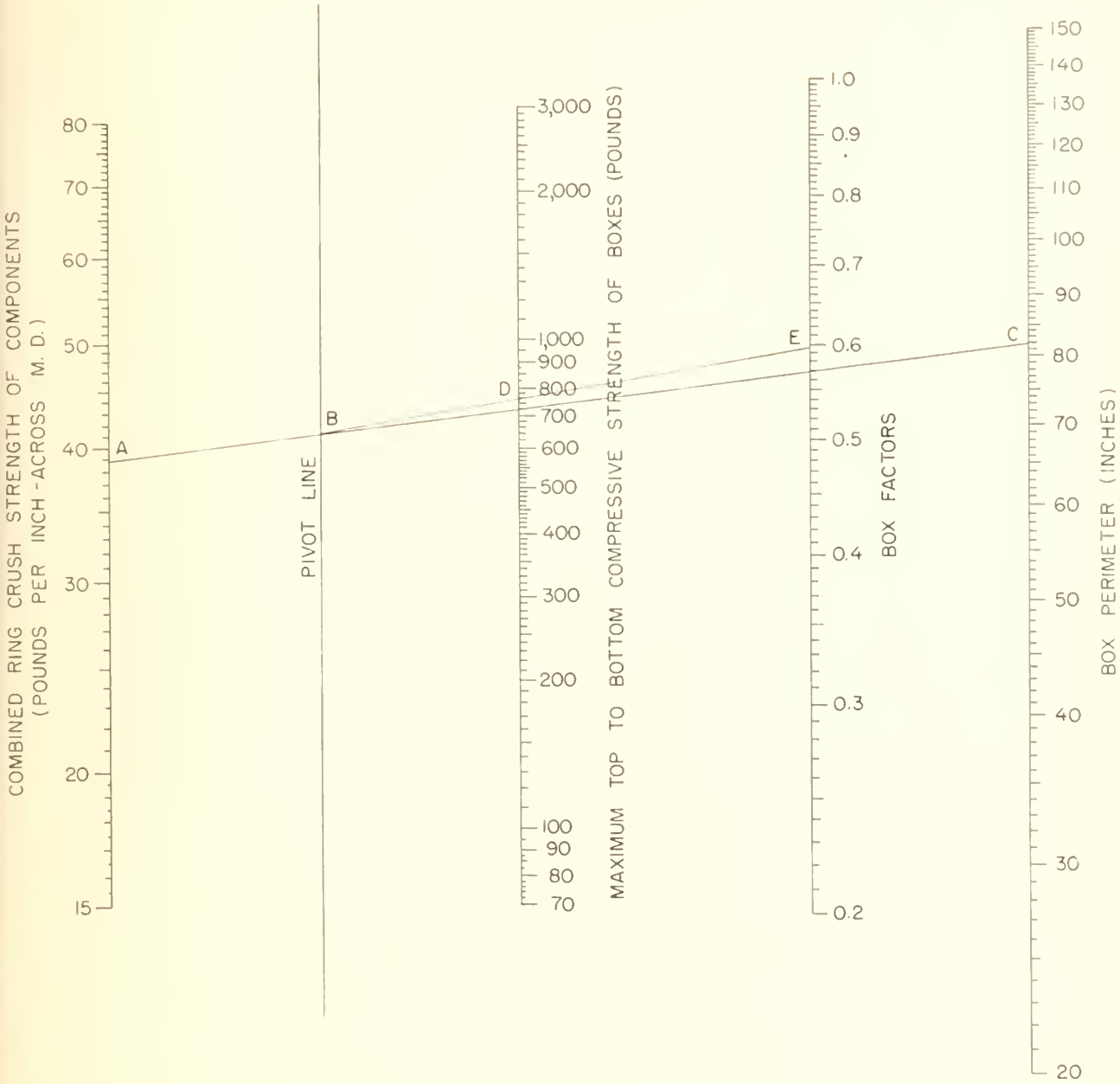
(ZM 88067 F)


COMBINED RING CRUSH STRENGTH OF COMPONENTS
(POUNDS PER INCH-ACROSS M. D.)
80
70
60
50
40
30
20
15
A
PIVOT LINE
B
MAXIMUM TOP TO BOTTOM COMPRESSIVE STRENGTH OF BOXES (POUNDS)
5,000
4,000
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
D
E
BOX FACTORS
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
C
BOX PERIMETER (INCHES)
150
140
130
120
110
100
90
80
70
60
50
40
30
20

M 88067 F

Figure 4.--Chart for determining the top-to-bottom compressive strength of B-flute corrugated fiberboard boxes when the flutes are vertical in the side walls. The resulting determinations will be for a box conditioned in the same atmosphere in which the ring-crush values were obtained.

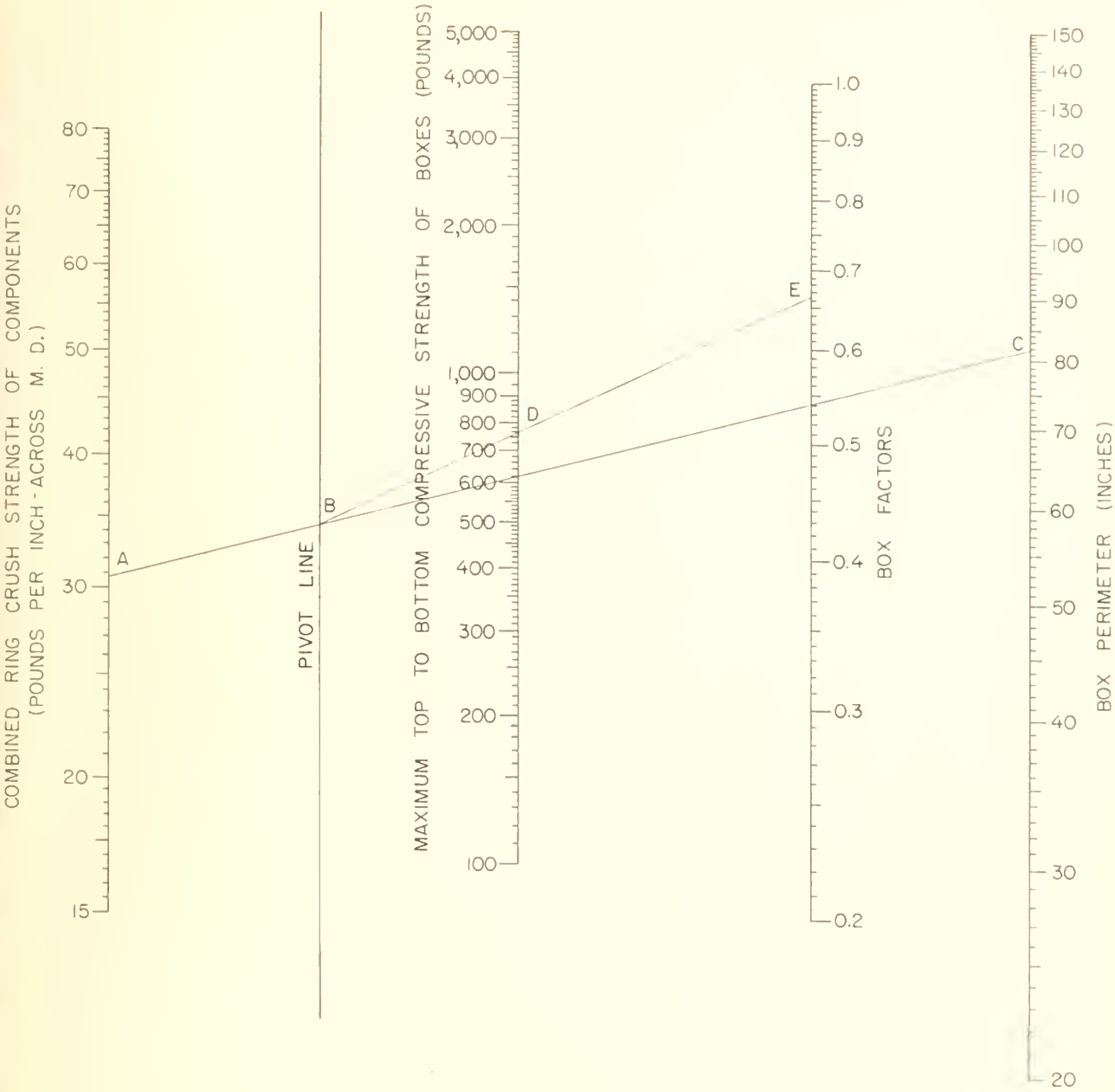
(ZM 88068 F)


COMBINED RING CRUSH STRENGTH OF COMPONENTS
(POUNDS PER INCH-ACROSS M. D.)
80
70
60
50
40
30
20
15
A
B
PIVOT LINE
D
MAXIMUM TOP TO BOTTOM COMPRESSIVE STRENGTH OF BOXES (POUNDS)
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
90
80
70
E
BOX FACTORS
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
C
BOX PERIMETER (INCHES)
150
140
130
120
110
100
90
80
70
60
50
40
30
20

Z M 88068 F

Figure 5.--Chart for determining the top-to-bottom compressive strength of C-flute corrugated fiberboard boxes when the flutes are vertical in the side walls. The resulting determinations will be for a box conditioned in the same atmosphere in which the ring-crush values were obtained.

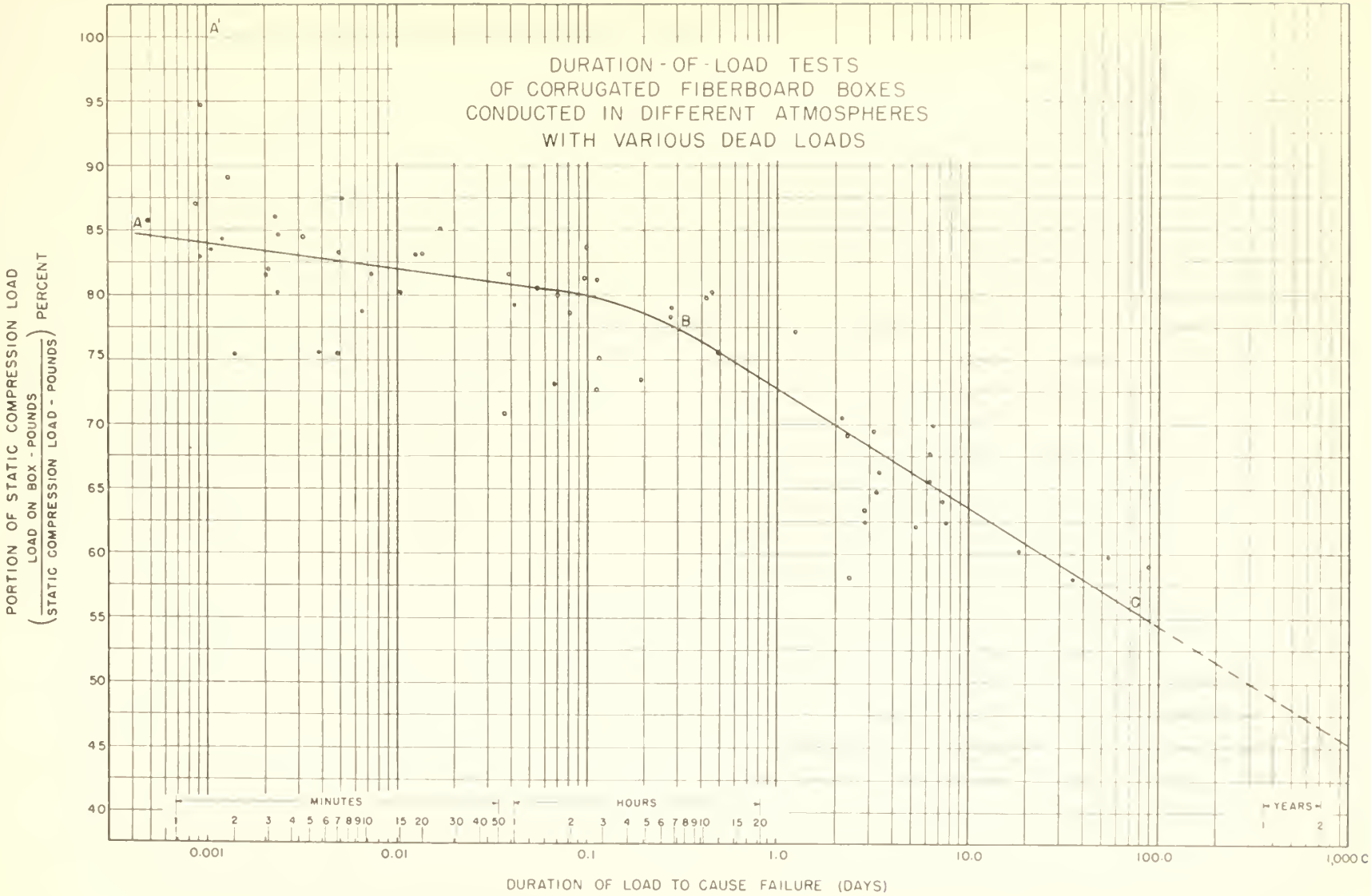
(ZM 88069 F)


COMBINED RING CRUSH STRENGTH OF COMPONENTS
(POUNDS PER INCH-ACROSS M. D.)
80
70
60
50
40
30
20
15
A
PIVOT LINE
B
MAXIMUM TOP TO BOTTOM COMPRESSIVE STRENGTH OF BOXES (POUNDS)
5,000
4,000
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
D
E
BOX FACTORS
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
C
BOX PERIMETER (INCHES)
150
140
130
120
110
100
90
80
70
60
50
40
30
20

Z M 88069 F

Figure 6.--Duration-of-load tests of corrugated fiberboard boxes conducted in different atmospheres with various dead loads.

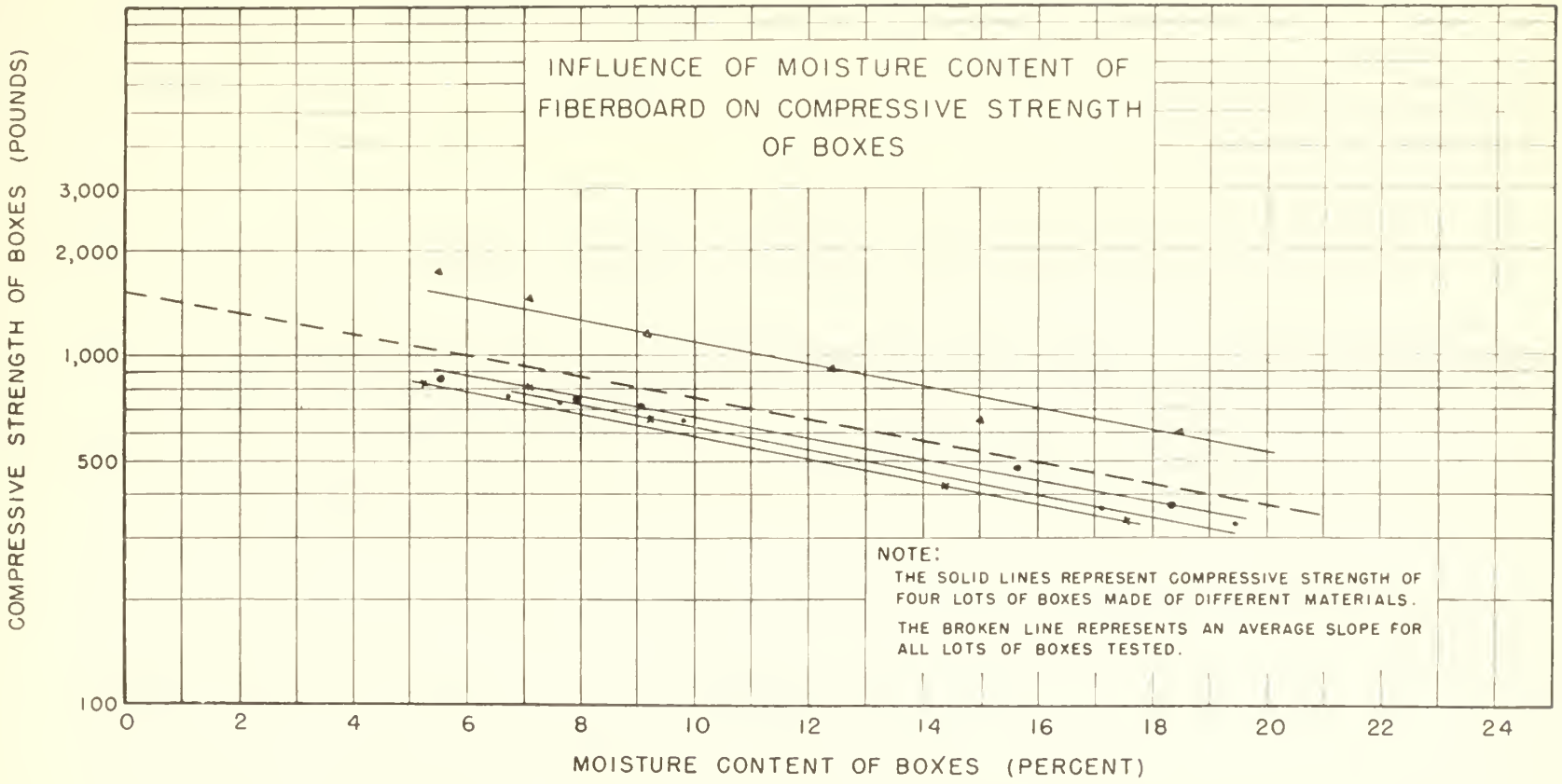
(ZM 87363 F)


DURATION-OF-LOAD TESTS
OF CORRUGATED FIBERBOARD BOXES
CONDUCTED IN DIFFERENT ATMOSPHERES
WITH VARIOUS DEAD LOADS
PORTION OF STATIC COMPRESSION LOAD
(LOAD ON BOX - POUNDS / STATIC COMPRESSION LOAD - POUNDS) PERCENT
100
95
90
85
80
75
70
65
60
55
50
45
40
A'
A
B
C
MINUTES
1 2 3 4 5 6 7 8 9 10 15 20 30 40 50
HOURS
1 2 3 4 5 6 7 8 9 10 15 20
YEARS
1 2
0.001
0.01
0.1
1.0
10.0
100.0
1,000 C
DURATION OF LOAD TO CAUSE FAILURE (DAYS)

ZM 87363 F

Figure 7.--Influence of moisture content of fiberboard on compressive strength of boxes.

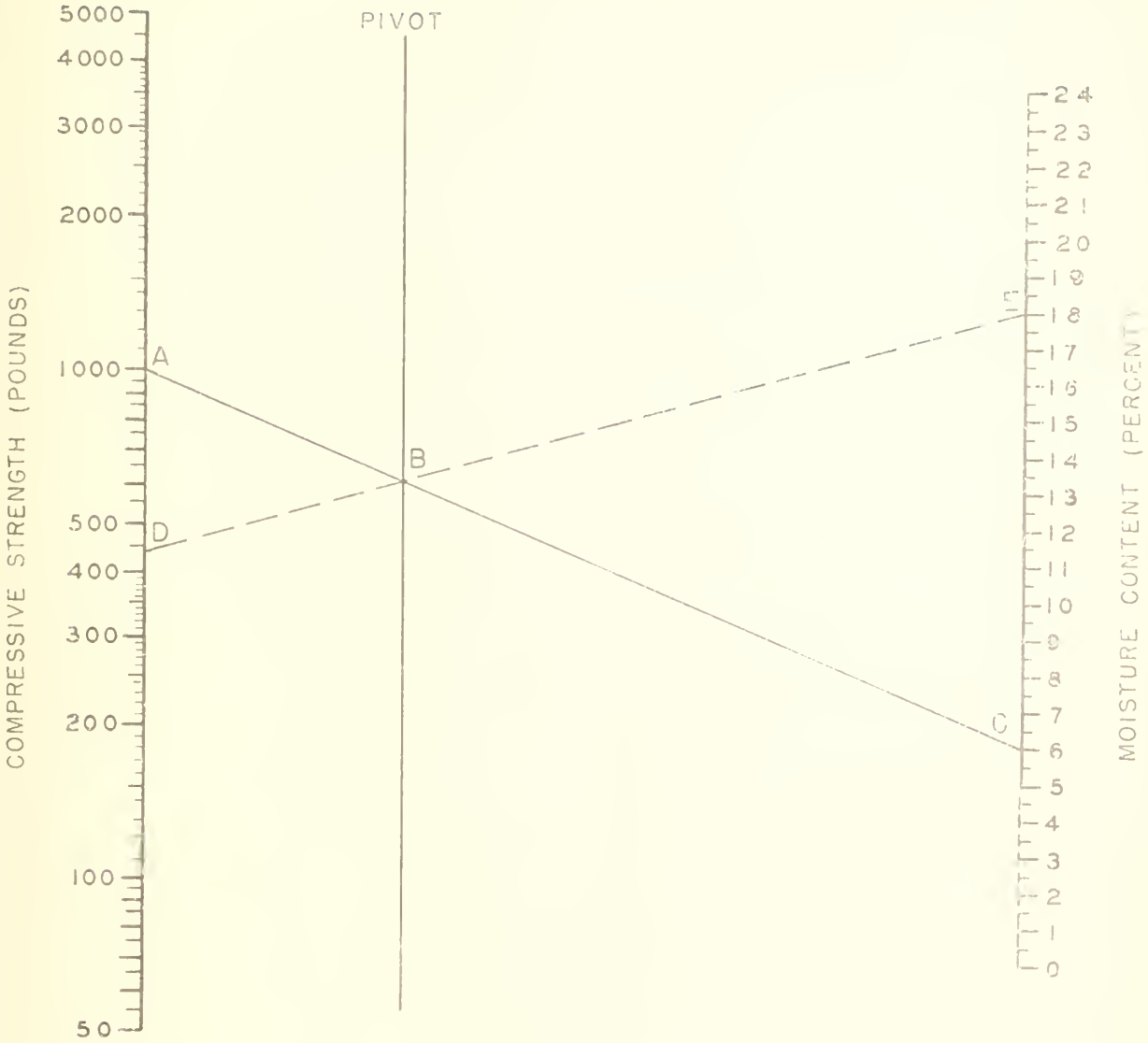
(ZM 87364 F)


INFLUENCE OF MOISTURE CONTENT OF FIBERBOARD ON COMPRESSIVE STRENGTH OF BOXES
COMPRESSIVE STRENGTH OF BOXES (POUNDS)
3,000
2,000
1,000
500
100
0
2
4
6
8
10
12
14
16
18
20
22
24
MOISTURE CONTENT OF BOXES (PERCENT)
NOTE:
THE SOLID LINES REPRESENT COMPRESSIVE STRENGTH OF FOUR LOTS OF BOXES MADE OF DIFFERENT MATERIALS.
THE BROKEN LINE REPRESENTS AN AVERAGE SLOPE FOR ALL LOTS OF BOXES TESTED.

ZN 87564 F

Figure 8.--Chart for converting top-to-bottom compressive strength of corrugated fiberboard boxes at one moisture content to strength at another moisture content.

(ZM 87365 F)


PIVOT
COMPRESSIVE STRENGTH (POUNDS)
5000
4000
3000
2000
1000
500
400
300
200
100
50
A
B
C
D
E
MOISTURE CONTENT (PERCENT)
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

ZM 87365 F